Linear Inequalities in Two Variables

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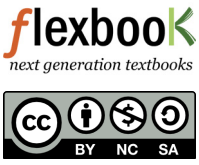

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CONCEPT 1

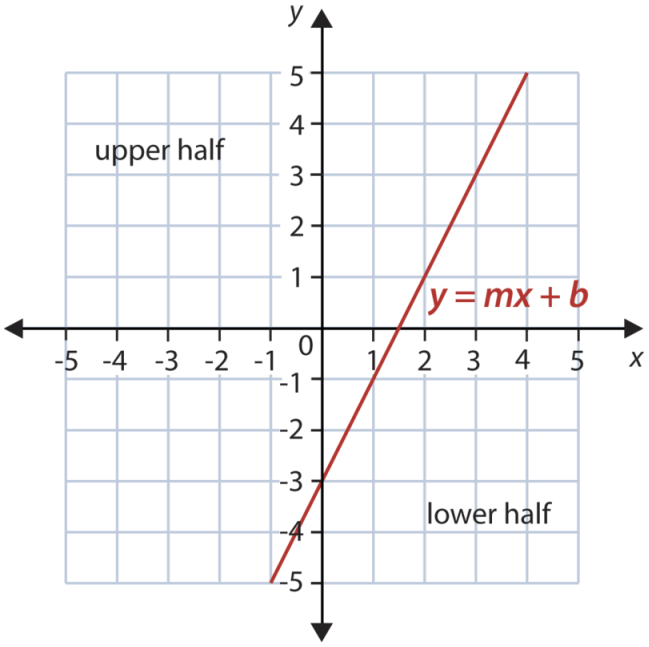
Linear Inequalities in Two Variables

Here you'll learn how to graph a linear inequality on a coordinate plane when the inequality has two variables.

Did you know that in European hockey leagues, a player gets 2 points for a goal and 1 point for an assist? Suppose a player's contract stipulates that he receives a bonus if he gets more than 100 points. What linear inequality could you write to represent this situation? How would you graph this inequality? In this Concept, you'll learn to graph linear inequalities in two variables so that you can properly analyze scenarios such as this one.

Guidance

When a linear equation is graphed in a coordinate plane, the line splits the plane into two pieces. Each piece is called a **half plane**. The diagram below shows how the half planes are formed when graphing a linear equation.

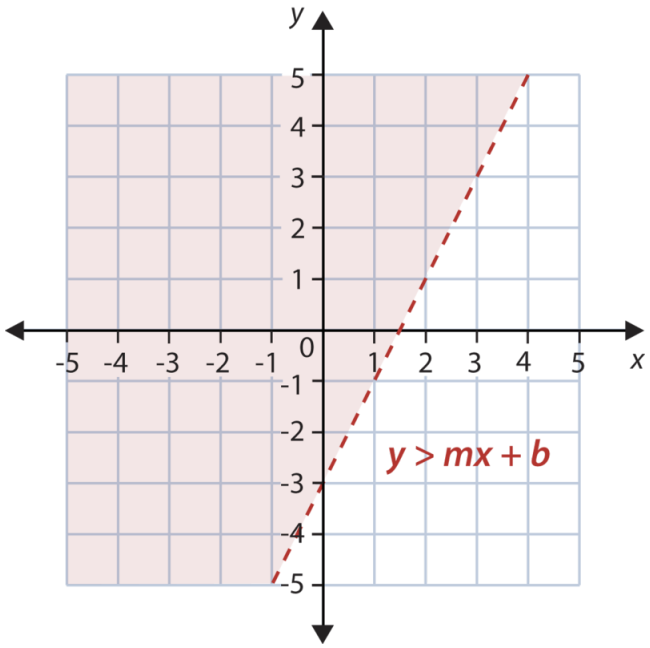


A linear inequality in two variables can also be graphed. Instead of graphing only the **boundary line** $(y = mx + b)$, you must also include all the other ordered pairs that could be solutions to the inequality. This is called the **solution set** and is shown by shading, or coloring, the half plane that includes the appropriate solutions.

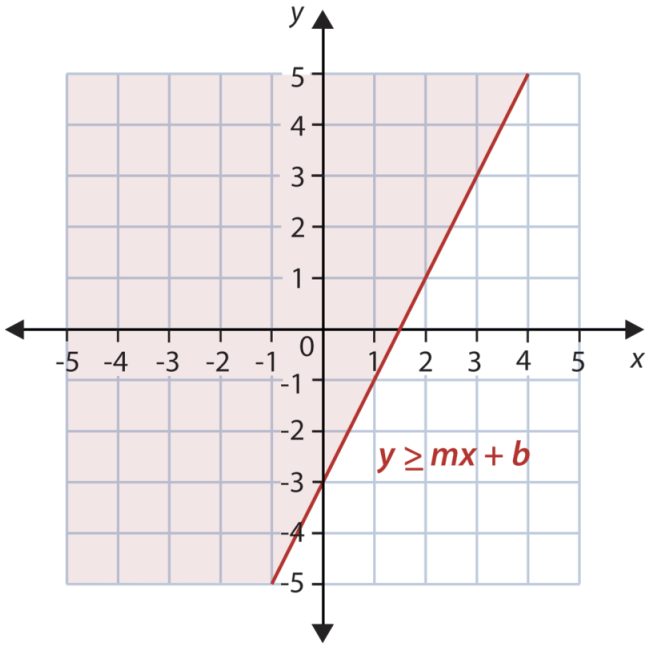
When graphing inequalities in two variables, you must remember when the value is included $(\leq or \geq)$ or not included $(< or >)$. To represent these inequalities on a coordinate plane, instead of shaded or unshaded circles, we use solid and dashed lines. We can tell which half of the plane the solution is by looking at the inequality sign.

- $>$ The solution is the half plane above the line.
- $\geq$ The solution is the half plane above the line and also all the points on the line.
- $<$ The solution is the half plane below the line.
- $\leq$ The solution is the half plane below the line and also all the points on the line.

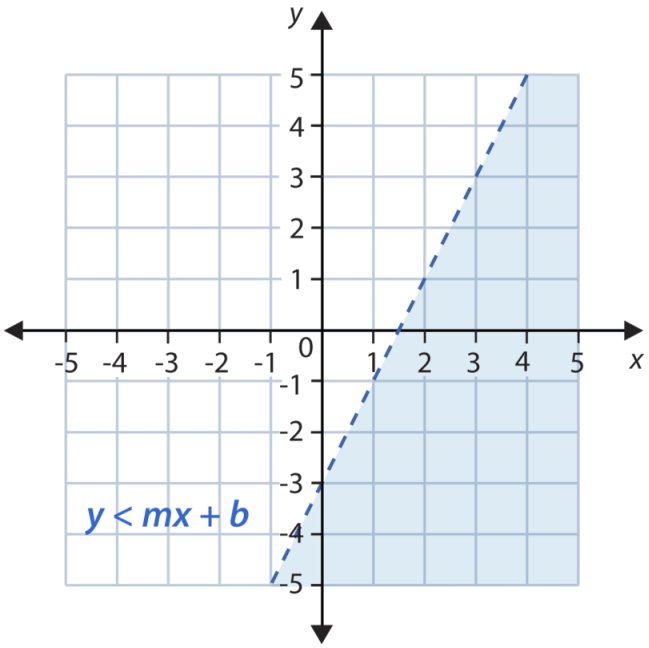
The solution of $y > mx + b$ is the half plane above the line. The dashed line shows that the points on the line are not part of the solution.



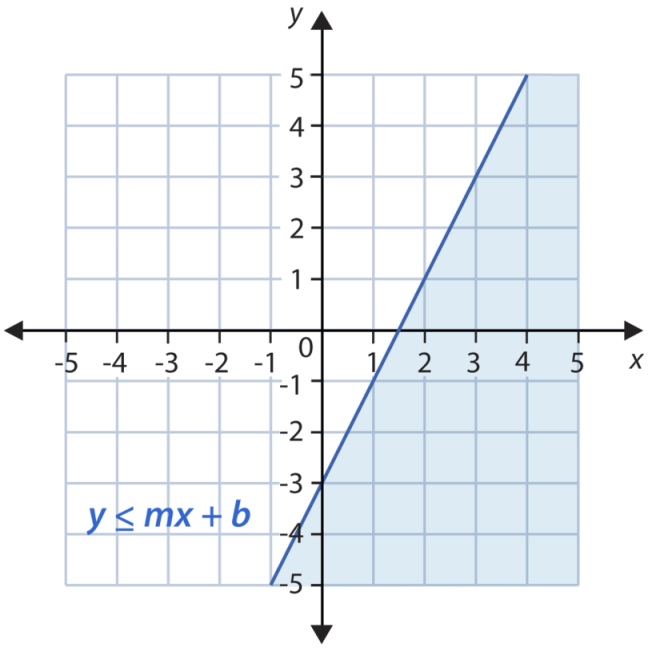
The solution of $y \geq mx + b$ is the half plane above the line and all the points on the line.



The solution of $y < mx + b$ is the half plane below the line.



The solution of $y \leq mx + b$ is the half plane below the line and all the points on the line.



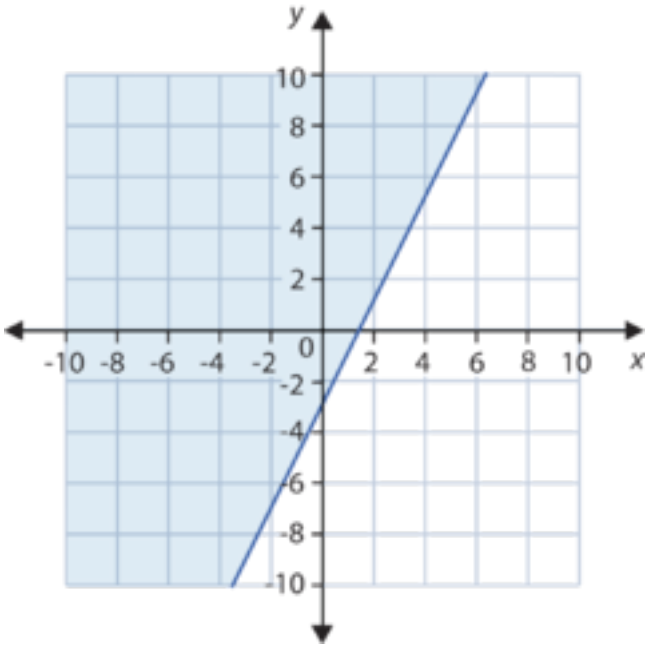
Example A

Graph the inequality $y \geq 2x - 3$.

Solution:

This inequality is in slope-intercept form. Begin by graphing the line. Then determine the half plane to color.

- The inequality is $\geq$, so the line is solid.
- According to the inequality, you should shade the half plane above the boundary line.



In general, the process used to graph a linear inequality in two variables is:

Step 1: Graph the equation using the most appropriate method.

- Slope-intercept form uses the $y-$intercept and slope to find the line.
- Standard form uses the intercepts to graph the line.
- Point-slope uses a point and the slope to graph the line.

Step 2: If the equal sign is not included, draw a dashed line. Draw a solid line if the equal sign is included.

Step 3: Shade the half plane above the line if the inequality is "greater than." Shade the half plane under the line if the inequality is "less than."

Example B

Julian has a job as an appliance salesman. He earns a commission of $60 for each washing machine he sells and $130 for each refrigerator he sells. How many washing machines and refrigerators must Julian sell to make $1,000 or more in commission?

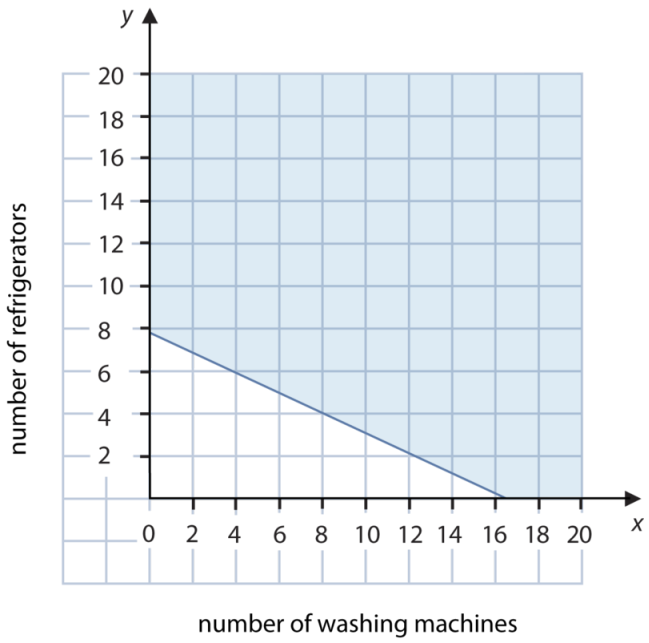
Solution:

Determine the appropriate variables for the unknown quantities. Let $x =$ number of washing machines Julian sells and let $y =$ number of refrigerators Julian sells.

Now translate the situation into an inequality. $60x + 130y \geq 1,000$.

Graph the standard form inequality using its intercepts. When $x = 0, y = 7.692$. When $y = 0, x = 16.667$. The line will be solid.

We want the ordered pairs that are solutions to Julian making more than $1,000, so we shade the half plane above the boundary line.



Graphing Horizontal and Vertical Linear Inequalities

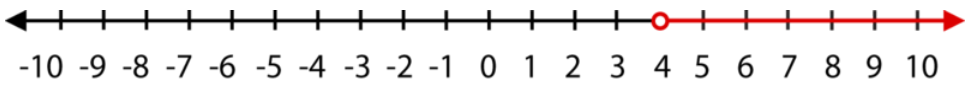
Linear inequalities in one variable can also be graphed in the coordinate plane. They take the form of horizontal and vertical lines; however, the process is identical to graphing **oblique**, or slanted, lines.

Example C

Graph the inequality $x > 4$ *on*: 1) *a number line and* 2) *the coordinate plane.*

Solution:

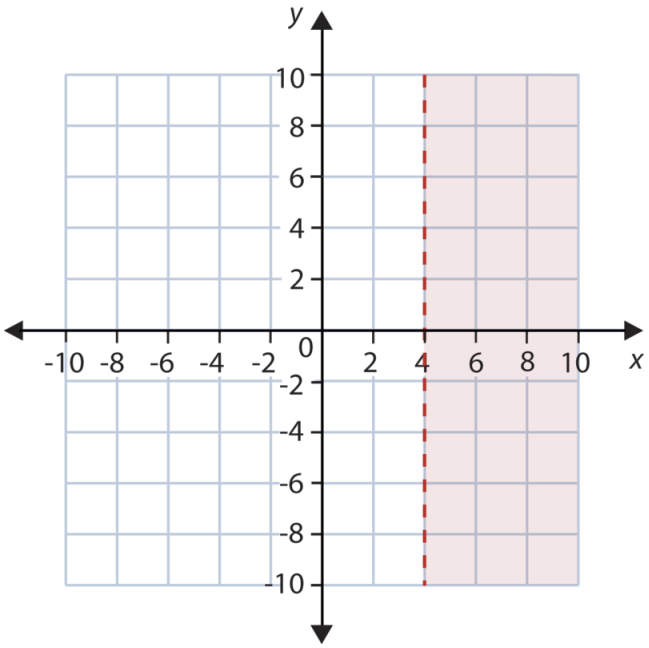
Remember what the solution to $x > 4$ looks like on a number line.



The solution to this inequality is the set of all real numbers x that are larger than four but not including four.

On a coordinate plane, the line $x = 4$ is a vertical line four units to the right of the origin. The inequality **does not equal** four, so the vertical line is dashed. This shows the reader that the ordered pairs on the vertical line $x = 4$ are not solutions to the inequality.

The inequality is looking for all $x-$coordinates larger than four. We then color the half plane to the right, symbolizing $x > 4$.



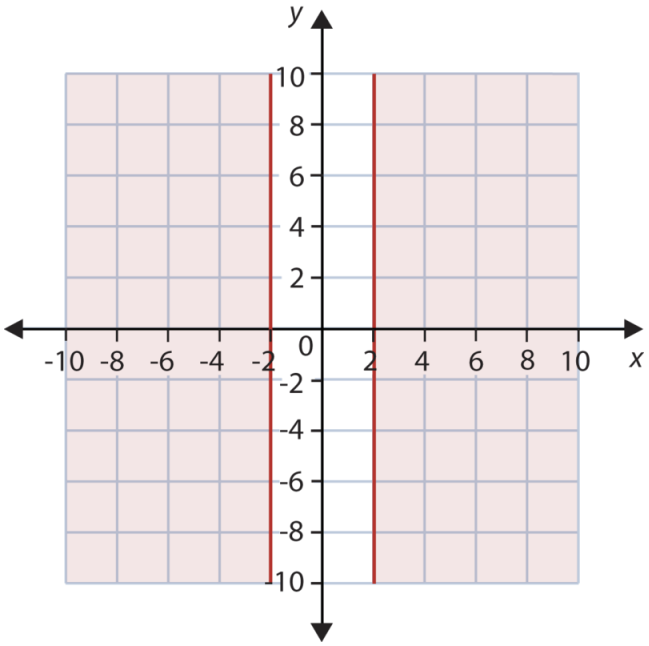
Graphing absolute value inequalities can also be done in the coordinate plane. To graph the inequality $|x| \geq 2$, we can recall a previous Concept and rewrite the absolute value inequality.

$x \leq -2$ or $x \geq 2$

Then graph each inequality on a coordinate plane.

In other words, the solution is all the coordinate points for which the value of x is smaller than or equal to –2 and greater than or equal to 2. The solution is represented by the plane to the left of the vertical line $x = -2$ and the plane to the right of line $x = 2$.

Both vertical lines are solid because points on the line are included in the solution.



Guided Practice

A pound of coffee blend is made by mixing two types of coffee beans. One type costs $9.00 per pound and another type costs $7.00 per pound. Find all the possible mixtures of weights of the two different coffee beans for which the blend costs $8.50 per pound or less.

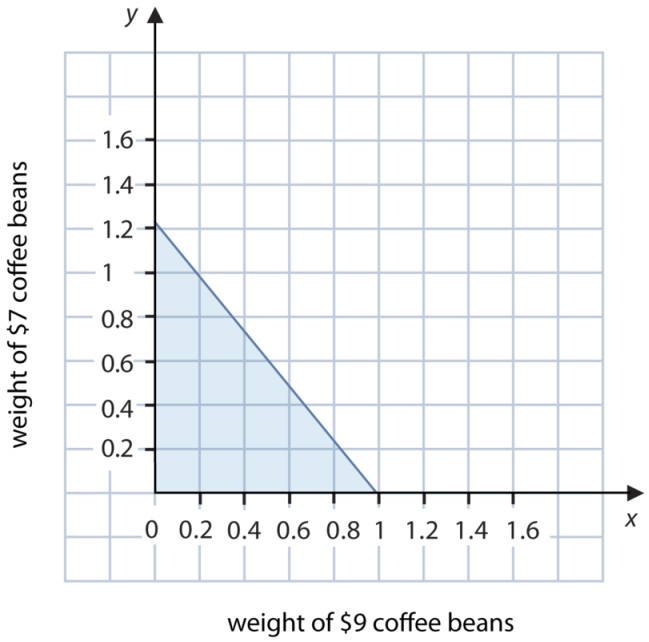
Solution:

Begin by determining the appropriate letters to represent the varying quantities.

Let $x =$ weight of \$9.00 per pound coffee beans in pounds and let $y =$ weight of \$7.00 per pound coffee beans in pounds.

Translate the information into an inequality. $9x + 7y \leq 8.50$.

Because the inequality is in standard form, it will be easier to graph using its intercepts.



When $x = 0, y = 1.21$. When $y = 0, x = 0.944$.

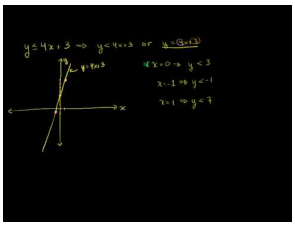
Graph the inequality. The line will be solid. We shade below the line.

We graphed only the first quadrant of the coordinate plane because neither bag should have a negative weight.

The blue-shaded region tells you all the possibilities of the two bean mixtures that will give a total less than or equal to \$8.50.

Practice

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra:Graphing Inequalities (8:03)



MEDIA

Click image to the left for more content.

1. Define *half plane.*
2. In which cases would the boundary line be represented by a dashed line when graphing an inequality?
3. In which cases would the boundary line be represented by a solid line when graphing an inequality?
4. What is a method to help you determine which half plane to color when graphing an inequality?

In 5 – 26, graph each inequality in a coordinate plane.

5. $x < 20$
6. $y \geq -5$
7. $y \leq 6$
8. $|x| > 10$
9. $|y| \leq 7$
10. $y \leq 4x + 3$
11. $y > -\frac{x}{2} - 6$
12. $y \leq -\frac{1}{2}x + 5$
13. $3x - 4y \geq 12$
14. $x + 7y < 5$
15. $y < -4x + 4$
16. $y > \frac{7}{2}x + 3$
17. $6x + 5y > 1$
18. $6x - 5y \leq 15$
19. $2x - y < 5$
20. $y + 5 \leq -4x + 10$
21. $x - \frac{1}{2}y \geq 5$
22. $y + 4 \leq -\frac{x}{3} + 5$
23. $5x - 2y > 4$
24. $30x + 5y < 100$
25. $y \geq -x$
26. $6x - y < 4$
27. Lili can make yarn ankle bracelets and wrist bracelets. She has 600 yards of yarn available. It takes 6 yards to make one wrist bracelet and 8 yards to make one ankle bracelet. Find all the possible combinations of ankle bracelets and wrist bracelets she can make without going over her available yarn.
28. An ounce of gold costs $670 and an ounce of silver costs $13. Find all possible weights of silver and gold that make an alloy (combination of metals) that costs less than $600 per ounce.
29. A phone company charges 50 cents per minute during the daytime and 10 cents per minute at night. How many daytime minutes and nighttime minutes would you have to use to pay more than $20.00 over a 24-hour period?
30. Jesu has $30 to spend on food for a class barbeque. Hot dogs cost $0.75 each (including the bun) and burgers cost $1.25 (including bun and salad). Plot a graph that shows all the combinations of hot dogs and burgers he could buy for the barbecue, spending less than $30.00.
31. At the local grocery store, strawberries cost $3.00 per pound and bananas cost $1.00 per pound. If I have $10 to spend between strawberries and bananas, draw a graph to show what combinations of each I can buy and spend at most $10.